Heavy diquark in baryons containing a single heavy quark and the weak form factors

Le Viet Dung*

International Centre for Theoretical Physics, Trieste 34100, Italy

Nguyen Ai Viet

Physics Department, Syracuse University, Syracuse, New York 13244

Abstract

It is shown that the number of independent weak form factors collapses if the heavy diquark exists inside the baryons containing a single heavy quark. The relations between the weak form factors are quite different in the case of light diquark. So a careful analysis on the future data of the weak form factors would clarify that in those baryons the correlation between the heavy quark and a light quark is stronger or weaker than the one between two light quarks.

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I. INTRODUCTION

In the limit the heavy quark mass going to infinity, a new $SU(2N_f)$ (N_f is number of heavy flavors) spin-flavor symmetry appears in QCD [1]. As a consequence, this symmetry implies that all the form factors involved in the semi-leptonic decays of B meson can be expressed by one single universal Isgur-Wise function. Recently [2], it has been argued that in the baryons containing a single heavy quark (hereafter we shall refer to as 1HQ-baryons) the correlation between the heavy quark and one light quark is much stronger than the one between two light quarks. That is to say, the presence of a heavy diquark could be more probable than the one of a light diquark in 1HQ-baryons. The main arguments are follows: The gluons that the heavy quark exchanges with light degrees of freedom are soft only for the heavy quark but are extremely hard for the light degrees of freedom (their momenta are of order $\sqrt{M_Q.\Lambda_{QCD}}$ [3]). Suppose that at some moment there exists some light diquark. The dynamics also contains hard gluon exchanges which have enough energy to break the binding of the light diquark, which is of order Λ_{QCD}. If we integrate the hard gluons into the diquark to have a strong binding of the diquark and a weak interaction of it with the heavy quark, we must have a very large diquark which would loose the sense of its own use. On the other hand, the heavy diquark must be very stable, since the hardest gluons have been integrated into the diquark to build a strong binding. The remaining light diquark has not enough energy to break this binding. The similar approximation has been applied in atomic physics: The He atom can be well approximated as a system of an electron with a He^+ ion. It could hardly be considered as a He^{++} ion and a Cooper pair of two electrons. Some calculations [3] have also indicated that in the 1HQ-baryons the correlation of qq is always much weaker than the one of Qq (q denotes light quarks, while Q denotes heavy quarks).

So far, we are no fanatics of the idea and of the heuristic arguments given above, unless it could be proved from the first principles of QCD. Such nonperturbative aspects of QCD are still exceedingly difficult to attack at present. We need more evidences to headaway the idea, which would lead to an interesting supersymmetry between 1HQ-mesons and baryons[2].

Fortunately, in this paper we are able to offer a method to analyze the weak decays of 1HQ-baryons, which may likely trace back to the presence of the existing diquark. We shall show that the diquark, if exists, would leave its traces on a series of relations between the weak form factors of baryons. Quite different relations are implied from the presumed existence of the heavy diquark or of the light diquark. So a careful analysis of the baryon's weak decay data in the future will clarify the dominant correlation in 1HQ-baryons.

In such an analysis a proper definition of diquark is very essential. Although the idea of diquark was suggested as early as in 1966 [4], there are still many contradictory definitions of diquark [5]. Many papers contain the assumption, sometimes hidden, that diquarks can be treated as elementary. Some others state that diquarks are quasi-elementary constituents of baryons, or even assume that diquarks are not elementary at all but rather correlated states of two quarks. In this paper we shall use another definition of diquark, which would satisfy the requirement of Lichtenberg [5]: a diquark is a correlated state of two quarks and the quarks inside and outside the diquark should be antisymmetrized. To give the diquark a sense we shall also assume that the diquark spin is a well-defined quantum number. This is the case when the spin-spin interaction between the diquark and the remaining degrees of freedom is small comparing to the color force. In such a strong regime of QCD, the diquark spin is a good quantum number (while the spin of different quark can be not a good quantum number). Therefore, we can split the spin part of the baryon wave functions respectively. In the large limit of the heavy quark mass, we can also postulate the heavy quark symmetries for 1HQ-baryons. So, the heavy diquark picture can be considered as a further step beyond the spectator quark model. Such an approximation is similar to the one when we approximate an atom by an electron bound to an ion neglecting the electron-electron interaction. We shall follow the covariant Bethe-Salpeter formalism [6] and show that the assumption of heavy diquark reduces the number of the weak form factors of 1HQ-baryons comparing to the cases where the light diquark or the heavy quark symmetry alone are supposed.

The paper is organized as follows: In Sec.II we shall briefly review the covariant Bethe-

Salpeter formalism and the weak current induced baryon transitions. In Sec.III giving a proper definition of diquark we develop the covariant Bethe-Salpeter formalism for the heavy diquark-quark systems to construct the wave functions of heavy baryons. The heavy to heavy and the heavy to light baryon transitions are discussed in Secs.IV and V, respectively. In Sec.VI we shall discuss the results and suggest some new possibilities opened by those ones. The light diquark-quark picture of 1HQ-baryons does not lead to any simplification since actually the spin decoupling of the heavy quark coincides to the one of the light diquark.

II. THE COVARIANT BETHE-SALPETER FORMALISM AND THE WEAK CURRENT INDUCED TRANSITIONS OF THE 1HQ-BARYONS

Within the covariant constituent quark model, the baryon Bethe-Salpeter amplitude is:

$$B =< 0|T(\psi_\alpha(x_1)\psi_\beta(x_2)\psi_\gamma(x_3)|B > \tag{2.1}$$

where the ψ's represent the quark fields and $|B >$ is a baryon state. The B-S amplitude contains the baryon-quark vertex function with quark and baryon legs. The Feynman rules are given by quark propagators and the truncated B-S amplitude [7].

Diagramatically, the B-S amplitude is represented in Fig.1.

Fig.1

The Bethe-Salpeter wavefuntions of baryons should be fully antisymmetrized under the interchange of any two quarks. So the Fourier transform of the fully antisymmetric baryon B-S wave function is represented as follows:

$$B_{A_i;B_j;C_k} = \frac{\epsilon_{ijk}}{\sqrt{6}} B_{(A;B;C)}(P;k_1,k_2) \tag{2.2}$$

where $A = (\alpha, a)$, etc. represents the Dirac and flavor indices α and a, respectively. The color labels are denoted by i, j, k. The variables k and P denote the quark and baryon momenta respectively. By the conservation law we can allow the baryon wave function having only three arguments k_1, k_2 and P. The spin-flavor part $B_{(ABC)}$ should be fully symmetric under the interchange of the indices A, B, C. By help of the spin-parity projectors we can project out the irreducible states [8,6] of spin $s = \frac{3}{2}$ and $s = \frac{1}{2}$.

The natural ansatz for the baryon Bethe-Salpeter wave functions in general is given as follows:

$$B_{(A;B;C)}(P; k_1, k_2) = \chi^B_{\rho\delta\sigma;abc} A^{\rho\delta\sigma}_{\alpha\beta\gamma}(P; k_1, k_2) \tag{2.3}$$

Here $A^{\rho\delta\sigma}_{\alpha\beta\gamma}(P; k_1, k_2)$ is a sixth-rank Dirac tensor function, while $\chi^B_{\rho\delta\sigma;abc}$ is the spin-parity flavor projector.

At the moment we ignore the flavor-dependent part having in mind that it is determined uniquely by the spin-dependent part and it can be constructed conveniently, whenever it will be necessary. In general, the spin-parity projectors for the s-wave baryons are given as follows [6]:

$$\chi^{\Lambda}_{\rho\delta\sigma} = u_\rho[(\not{v} + 1)\gamma_5 C]_{\delta\sigma} \tag{2.4}$$

$$\chi^{\Sigma}_{\rho\delta\sigma} = -[((\gamma^\mu + v^\mu)\gamma_5 u]_\rho[(\not{v} + 1)\gamma_\mu C]_{\delta\sigma} \tag{2.5}$$

$$\chi^{\Sigma^*}_{\rho\delta\sigma} = u^\mu_\rho[(\not{v} + 1)\gamma_\mu C]_{\delta\sigma} \tag{2.6}$$

where $v_\mu = P_\mu / M_B$ is the four-velocity of baryons. The Dirac spinors u_α satisfy the equation:

$$(\not{v} - 1)u = 0 \tag{2.7}$$

The Rarita-Schwinger spinors u^μ_α satisfy the equations:

$$(\not{v} - 1)u^\mu \tag{2.8}$$

$$\gamma_\mu u^\mu = 0 \tag{2.9}$$

$$v_\mu u^\mu = 0 \tag{2.10}$$

The general wave functions given in the formula (2.3) can be used to describe the light baryons. For the 1HQ-baryons, it has been shown that if one of quarks becomes heavy, in the limit of the heavy quark mass going to infinity, we have the following good approximation:

$$A^{\rho\delta\sigma}_{\alpha\beta\gamma} \approx \delta^\rho_\alpha A^{\delta\sigma}_{\beta\gamma}(v, k_1, k_2) \tag{2.11}$$

Diagramatically, we can represent the 1HQ-baryon- quark Bethe-Salpeter amplitude as in Fig.2.

Fig.2

Consider the weak current induced baryon transitions, which are represented diagramatically in Fig.3.

Fig.3

The Feynman rules are given by the quark propagators and the truncated quark-baryon Bethe-Salpeter amplitudes as vertex functions. Therefore, the decay matrix element can be written as

$$< B_2(P_2)|J^{V-A}_\lambda|B_1(P_1) >= \int d^4k_1 d^4k_2 \bar{B}^{\alpha\beta\gamma}[\gamma_\lambda(1-\gamma_5)]^{\alpha'}_\alpha (\not{k}_1 - m_1)^{\beta'}_\beta \tag{2.12}$$

$$(\not{k}_2 - m_2)^{\gamma'}_\gamma B_{\alpha'\beta'\gamma'}.b(B_1, B_2) \tag{2.13}$$

We have already absorbed the Kobayashi-Maskawa-Cabbibo matrix element into the trace of the flavor part $b(B_1, B_2)$. Having an ansatz of the wave functions we can compute the decay matrix for different transitions. The results are follows (see Refs. [7] for more details).

A. *The heavy to heavy baryon transitions*

Using the baryon Bethe-Salpeter wave function projected out by the spin-parity projectors (2.4-6) and the approximation (2.11) we can write down the decay matrix elements as follows:

i) $\Lambda(\Xi)$-type $\frac{1}{2}^+ \to \frac{1}{2}^+$ baryon transitions

$$< \Lambda_2(P_2)|J_\lambda^{V-A}|\Lambda_1(P_1) >= b(\Lambda_1, \Lambda_2)\bar{u}_2(P_2)\gamma_\lambda(1-\gamma_5)u_1(P_1).F_\Lambda \tag{2.14}$$

ii) $\Sigma(\Omega)$-type $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions

$$\begin{aligned}< \Sigma_2(P_2)|J_\lambda^{V-A}|\Sigma_1(P_1) > &= b(\Sigma_1, \Sigma_2)\bar{u}_2(P_2)\gamma_5(\gamma^\mu + v_2^\mu)\gamma_\lambda(1-\gamma_5)(\gamma^\nu + v_1^\nu) \\ &\gamma_5 u_1(P_1)(F_1 g_{\mu\nu} + F_2 v_{1\mu}v_{2\nu}) = 2b(\Sigma_1, \Sigma_2)\bar{u}_2(P_2)\Big[F_L\gamma_\lambda(1-\gamma_5) \\ -\frac{2}{1+\omega}(F_L + F_T)(v_{1\lambda} + v_{2\lambda}) &+ \frac{2}{1-\omega}(F_L - F_T)(v_{2\lambda} - v_{1\lambda})\gamma_5\Big]u_1(P_1)\end{aligned} \tag{2.15}$$

iii) $\Sigma(\Omega) \to \Sigma^*(\Omega^*)$-type $\frac{1}{2}^+ \to \frac{3}{2}^+$ transitions

$$\begin{aligned}< \Sigma_2^*(P_2)|J_\lambda^{V-A}|\Sigma_1(P_1) > &= b(\Sigma_1, \Sigma_2^*)\bar{u}_2^\mu\gamma_\lambda(1-\gamma_5)(\gamma^\nu + v_1^\nu)\gamma_5 u_1(P_1)\Big(F_1 g_{\mu\nu} + F_2 v_{1\mu}v_{2\nu}\Big) \\ &= b(\Sigma_1, \Sigma_2^*)\bar{u}_2^\mu(P_2)\Big[F_T g_{\mu\lambda}(1+\gamma_5) + \frac{1}{2(1+\omega)}(F_L + F_T)v_{1\mu}\gamma_\lambda\gamma_5 \\ &- \frac{1}{1-\omega^2}(F_L - \omega F_T)v_{1\mu}v_{2\lambda}(1+\gamma_5)\Big]u_1(P_1)\end{aligned} \tag{2.16}$$

iv) $\Sigma^*(\Omega^*) \to \Sigma^*(\Omega^*)$-type $\frac{3}{2}^+ \to \frac{3}{2}^+$ transitions

$$\begin{aligned}&< \Sigma_2^*(P_2)|J_\lambda^{V-A}|\Sigma_2^*(P_1) > \\ &= b(\Sigma_1^*, \Sigma_2^*)\bar{u}_2^\mu(P_2)\gamma_\lambda(1-\gamma_5)u_1^\nu(P_1)(F_1 g_{\mu\nu} + F_2 v_{1\mu}v_{2\nu}) \\ &= 4b(\Sigma_1^*, \Sigma_2^*)\bar{u}_2^\mu\gamma_\lambda(1-\gamma_5)u_1^\nu(P_1)\{-F_T g_{\mu\nu} + \frac{1}{1-\omega^2}(F_L - \omega F_T)v_{1\mu}v_{2\nu}\}\end{aligned} \tag{2.17}$$

where $\omega = v_1.v_2$ is the new variable used instead of $-q^2 = (P_1 - P_2)^2$. The form factors F_L, F_T are combinations of F_1, F_2:

$$F_L = \omega F_1 + (1-\omega^2)F_2$$
$$F_T = F_2 \tag{2.18}$$

The invariant form factors F_Λ, F_1 and F_2 are given by the loop integrals

$$F_\Lambda(\omega) = \int d^4k_1 d^4k_2 [(\not{v}_2+1)\gamma_5 C]^{+\delta\sigma} A^{+\beta\gamma}{}_{\delta\sigma}(v_2,k_1,k_2)$$
$$(\not{k}_1 - m_1)^{\beta'}_{\beta} (\not{k}_2 - m_2)^{\gamma'}_{\gamma} A^{\delta'\sigma'}_{\beta'\gamma'}(v_1,k_1,k_2)[(\not{v}_1+1)\gamma_5 C]_{\delta'\sigma'} \tag{2.19}$$
$$F_1(\omega)g_{\mu\nu} + F_2(\omega)v_{1\mu}v_{2\nu} = \int d^4k_1 d^4k_2 [(\not{v}_2+1)\gamma_\mu C]^{+\delta\sigma} A^{+\beta\gamma}{}_{\delta\sigma}(v_2,k_1,k_2)$$
$$(\not{k}_1 - m_1)^{\beta'}_{\beta} (\not{k}_2 - m_2)^{\gamma'}_{\gamma} A^{\delta'\sigma'}_{\beta'\gamma'}(v_1,k_1,k_2)[(\not{v}_1+1)\gamma_\nu C]_{\delta'\sigma'} \tag{2.20}$$

The translational invariance insures that the form factors should depend only on the variable ω (or q^2).

B. *The heavy to light baryon transitions*

Using the general ansatz (2.3) for light baryon wave functions and the heavy baryon wave functions we can calculate the following decay matrix elements:

i) $\Lambda_Q(\Xi_Q) \to \Lambda_q(\Xi_q)$, $\frac{1}{2}^+ \to \frac{1}{2}^+$ baryon transitions

$$< \Lambda_q(P_2)|J^{V-A}_\lambda|\Lambda_Q(P_1) >= b(\Lambda_q,\Lambda_Q)\bar{u}_2(P_2)[F^1_\Lambda + \not{v}_1 F^2_\Lambda]\gamma_\lambda(1-\gamma_5)u_1(P_1).F_\Lambda \tag{2.21}$$

where

$$\left(F^1_\Lambda + \not{v}_1 F^2_\Lambda + \not{v}_2 F^3_\Lambda + \not{v}_2 \not{v}_1 F^4_\Lambda\right)^\alpha_\rho = \int d^4k_1 d^4k_2 [(\not{v}_2+1)\gamma_5 C]^{+\delta\sigma} A^{+\alpha\beta\gamma}{}_{\rho\delta\sigma}(v_2,k_1,k_2)$$
$$(\not{k}_1 - m_1)^{\beta'}_{\beta} (\not{k}_2 - m_2)^{\gamma'}_{\gamma} A^{\delta'\sigma'}_{\beta'\gamma'}(v_1,k_1,k_2)[(\not{v}_1+1)\gamma_5 C]_{\delta'\sigma'} \tag{2.22}$$

The terms containing $\not{v}_2$ and $\not{v}_2\ \not{v}_1$ collapse because of the action on the spinor $\bar{u}_2$.

ii) $\Sigma_Q(\Omega_Q) \to \Sigma_q(\Omega_q)$, $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions

$$< \Sigma_q(P_2)|J_\lambda^{V-A}|\Sigma_Q(P_1) >$$

$$\begin{aligned}
&= b(\Sigma_Q,\Sigma_q)\bar{u}_2(P_2)\gamma_5(\gamma^\mu+v_2^\mu)L_{\mu\nu}\gamma_\lambda(1-\gamma_5)(\gamma^\nu+v_1^\nu)\gamma_5 u_1(P_1)\\
&= 2b(\Sigma_Q,\Sigma_q)\bar{u}_2(P_2)\gamma_5[\bar{F}_1\gamma_\nu+\bar{F}_2 v_{2\nu}+\bar{F}_3\gamma_\nu \not{v}_1\\
&+ \bar{F}_4 v_{2\nu} \not{v}_1]\gamma_\lambda(1-\gamma_5)(\gamma^\nu+v_1^\nu)\gamma_5 u_1(P_1) \qquad (2.23)
\end{aligned}$$

where

$$(L_{\mu\nu})^\alpha_\rho = \int d^4k_1 d^4k_2[(\not{v}_2+1)\gamma_\mu C]^{+\delta\sigma}A^{+\alpha\beta\gamma}_{\rho\delta\sigma}(v_2,k_1,k_2)$$
$$.\ (\not{k}_1-m_1)^{\beta'}_\beta.(\not{k}_2-m_2)^{\gamma'}_\gamma A^{\delta'\sigma'}_{\beta'\gamma'}(v_1,k_1,k_2)[(\not{v}_1+1)\gamma_\nu C]_{\delta'\sigma'} \qquad (2.24)$$

iii) $\Sigma_Q(\Omega_Q)\to\Sigma^*_q(\Omega^*_q)$, $\frac{1}{2}^+\to\frac{3}{2}^+$ transitions

$$< \Sigma^*_q(P_2)|J_\lambda^{V-A}|\Sigma_Q(P_1) >= b(\Sigma_Q,\Sigma^*_q)\bar{u}^\mu_2 L_{\mu\nu}\gamma_\lambda(1-\gamma_5)(\gamma^\nu+v_1^\nu)\gamma_5 u_1(P_1)$$

$$\begin{aligned}
&= b(\Sigma_q,\Sigma^*_Q)\bar{u}^\mu_2(P_2)\Big[-G^*_1 g_{\mu\nu}+G^*_2 v_{1\mu}v_{2\nu}+G^*_3 g_{\mu\nu} \not{v}_1+G^*_4 v_{1\mu}v_{2\nu} \not{v}_1\\
&+ G^*_5 v_{1\mu}\gamma_\nu+G^*_6 v_{1\mu}\gamma_\nu \not{v}_1\Big]\gamma_\lambda(1-\gamma_5)(\gamma^\nu+v_1^\nu)\gamma_5 u_1(P_1) \qquad (2.25)
\end{aligned}$$

iv) $\Sigma^*_Q(\Omega^*_Q)\to\Sigma^*_Q(\Omega^*_Q)$-type $\frac{3}{2}^+\to\frac{3}{2}^+$ transition

$$< \Sigma^*_Q(P_2)|J_\lambda^{V-A}|\Sigma^*_Q(P_1) >= b(\Sigma^*_1,\Sigma^*_2)\ \not{u}^\mu_2(P_2)L_{\mu\nu}\gamma_\lambda(1-\gamma_5)u^\nu_1(P_1)$$

$$\begin{aligned}
&= 4b(\Sigma^*_Q,\Sigma^*_Q)\ \not{u}^\mu_2(P_2)\Big[(-G^*_1 g_{\mu\nu}+G^*_2 v_{1\mu}v_{2\nu})\gamma_\lambda(1-\gamma_5)\\
&- (G^*_3 g_{\mu\nu}+G^*_4 v_{1\mu}v_{2\nu})\gamma_\lambda(1+\gamma_5)+2(G^*_3 g_{\mu\nu}+G^*_4 v_{1\mu}v_{2\nu})v_{1\lambda}(1-\gamma_5)\\
&+ 2G^*_5 v_{1\mu}g_{\nu\lambda}(1-\gamma_5)+2G^*_6 v_{1\mu}g_{\nu\lambda}(1+\gamma_5)\Big]u^\nu_1(P_1) \qquad (2.26)
\end{aligned}$$

III. THE COVARIANT BETHE-SALPETER FORMALISM FOR THE QUARK-DIQUARK SYSTEMS

The presence of diquark gives more restrictions to the ansatz given above. Let us define the concept of diquark in our framework. In order to have a sense, the diquark must have a well-defined spin. The spin-spin interaction must be negligible beside the static color potential between the diquark and the quark to preserve spin as a good quantum number.

Mathematically speaking, the diquark-quark picture is an approximation, where the spin part of the baryon wave function can be approximated as follows:

$$A^{\rho\delta\sigma}_{\alpha\beta\gamma} \approx A^{\rho\delta}_{\alpha\beta}(v, k_2) D^{\sigma}_{\gamma}(v, k_1) \tag{3.1}$$

Hereafter we shall use the variable v instead of P. Diagramatically, the quark-diquark Bethe-Salpeter wave functions are represented in Fig.4

Fig.4

Let us notice that the k_1-dependence of the diquark propagator and of the truncated diquark-quark B-S vertex function has been absorbed into $D(v, k_1)$. So, the light baryon Bethe-Salpeter wave functions are written in the form:

i) for the $\Lambda(\xi)$-type baryons ($J^P = \frac{1}{2}^+$)

$$\Lambda^l_{\alpha\beta\gamma} = u_\rho(v)[(\not{v} + 1)\gamma_5 C]_{\delta\sigma} A^{[\rho\delta]}_{\alpha\beta}(v, k_2) D^{\sigma}_{\gamma}(v, k_1) \tag{3.2}$$

ii) for the $\Sigma(\Omega)$-type baryons ($J^P = \frac{1}{2}^+$):

$$\Sigma^l_{\alpha\beta\gamma} = -[(\gamma^\mu + v^\mu)\gamma_5 u]_\rho [(\not{v} + 1)\gamma_\mu C]_{\delta\sigma} A^{(\rho\delta)}_{\alpha\beta}(v, k_2) D^{\sigma}_{\gamma}(v, k_1) \tag{3.3}$$

iii) for the $\Sigma^*(\Omega^*)$-type baryons ($J^P = \frac{3}{2}^+$)

$$\Sigma^{*l} = u^\mu_\rho(v)[(\not{v} + 1)\gamma_\mu C]_{\delta\sigma}.A^{(\rho\delta)}_{\alpha\beta}(v, k_2).D^{\sigma}_{\gamma}(v, k_1) \tag{3.4}$$

Specially, if one of quarks becomes heavy, the limit of the heavy quark mass going to infinity will give further simplification to the above ansatz. Let us consider two different cases

A. *The heavy diquark picture of 1HQ-baryons:*

This case is achieved by applying both the approximation (3.1) and (2.11). So, we have

$$A^{\rho\delta\sigma}_{\alpha\beta\gamma} \approx \delta^{\rho}_{\alpha} A^{\delta}_{\beta}(v, k_2) D^{\sigma}_{\gamma}(v, k_1) \tag{3.5}$$

Diagramatically, we represent the Bethe-Salpeter amplitude in Fig.5

Fig.5

We can also use the spin-parity projectors given in (2.4-6) to project out the baryon B-S wave functions in a similar way as in (3.2-5).

B. *The light diquark picture of 1HQ-baryons:*

Assuming that two light quarks of a 1HQ-baryon form a light diquark, if we split the spin part of the wave function accordingly, the B-S amplitude of the light diquark-heavy quark system is also represented by the diagram in Fig.3. It means that the existence of light diquark in 1HQ-baryons does not lead to any simplification.

IV. THE HEAVY TO HEAVY CURRENT INDUCED BARYON TRANSITIONS IN THE HEAVY DIQUARK MODEL

Diagramatically, these transitions are represented by the Feynman graph in Fig.6

Fig.6

Using the wave functions projected out by the spin-parity projectors (2.4-6) and the approximation (3.5) let us compute the decay matrix elements for the heavy to heavy baryon transitions.

The Bethe-Salpeter wave functions for baryons in the heavy diquark model are follows

$$\Lambda_{Q\alpha\beta\gamma} = u_\alpha(v)[(\not v+1)\gamma_5 C]_{\delta\sigma} A^\delta_\beta(v,k_2) D^\sigma_\gamma(v,k_1) \tag{4.1}$$

$$\Sigma_{Q\alpha\beta\gamma} = -[(\gamma^\mu + v^\mu)\gamma_5 u]_\alpha [(\not v+1)\gamma_\mu C]_{\delta\sigma} A^\delta_\beta(v,k_2) D^\sigma_\gamma(v,k_1) \tag{4.2}$$

$$\Sigma^*_{Q\alpha\beta\gamma} = u^\mu_\alpha(v)[(\not v+1)\gamma_\mu C]_{\delta\sigma} A^\delta_\beta(v,k_2) D^\sigma_\gamma(v,k_1) \tag{4.3}$$

Let us define the tensor:

$$\begin{aligned} A^{\delta'}_\delta(v_1,v_2) &= \int d^4k_2 A^{+\beta}_{\ \delta}(v_2,k_2)(\not k_2 - m_2)^{\beta'}_\beta A^{\delta'}_{\beta'}(v_1,k_2) \\ &= (A_1(\omega) + A_2(\omega)\ \not v_1 + A_3(\omega)\ \not v_2\ \not v_1 + A_4(\omega)\ \not v_2)^{\delta'}_\delta \end{aligned} \tag{4.4}$$

$$\begin{aligned} D^{\sigma'}_\sigma &= \int d^4k_1 D^{+\gamma}_{\ \sigma}(\not v_1 - m_1)^{\gamma'}_\gamma D^{\sigma'}_{\gamma'} \\ &= (D_1(\omega) + D_2(\omega)\ \not v_1 + D_3(\omega)\ \not v_2\ \not v_1 + D_4(\omega)\ \not v_2)^{\sigma'}_\sigma \end{aligned} \tag{4.5}$$

The matrix elements are computed as follows:

i) $\Lambda_Q(\Xi_Q) \to \Lambda_Q(\Xi_Q)$, $\frac{1}{2}^+ \to \frac{1}{2}^+$ baryon transitions

$$< \Lambda_1(P_2)|J^{V-A}_\lambda|\Lambda_(P_1) >= b(\Lambda_1,\Lambda_2)\gamma_\lambda(1-\gamma_5)u_1(P_1).\tilde F_\Lambda \tag{4.6}$$

where

$$\begin{aligned} \tilde F_\Lambda(\omega) &= [(\not v_2+1)\gamma_5 C]^{+\delta\sigma}[(\not v_1+1)\gamma_5 C]_{\delta'\sigma'} A^{\delta'}_\delta D^{\sigma'}_\sigma \\ &= Tr([(\not v_2+1)\gamma_5 C]^+[(\not v_1+1)\gamma_5 C])A_1(\omega)D_1(\omega) \\ &= 8A_1(\omega)D_1(\omega) \end{aligned} \tag{4.7}$$

ii) $\Sigma_Q(\Omega_Q) \to \Sigma_Q(\Omega_Q)$, $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions

$$\begin{aligned} &< \Sigma_Q(P_2)|J^{V-A}_\lambda|\Sigma_Q(P_1) > \\ &= \ b(\Sigma_Q,\Sigma_Q)\bar u_2(P_2)\gamma_5(\gamma^\mu + v_2^\mu)\gamma_\lambda(1-\gamma_5)(\gamma^\nu + v_1^\nu)\gamma_5 u_1(P_1)\tilde L_{\mu\nu} \end{aligned} \tag{4.8}$$

where

$$\tilde{L}_{\mu\nu} = [(\not{v}_2+1)\gamma_\mu C]^{\delta\sigma}[(\not{v}_1+1)\gamma_\nu C]_{\delta'\sigma'}A^{\delta'}_{\delta}D^{\sigma'}_{\sigma}$$
$$= Tr([(\not{v}_2+1)\gamma_\mu c]^{+}[(\not{v}_1)\gamma_\nu C])A_1(\omega)D_1(\omega)$$
$$= 8(g_{\mu\nu}-v_{1\mu}v_{2\nu})A_1(\omega)D_1(\omega) \tag{4.9}$$

iii) $\Sigma_Q(\Omega_Q)\to\Sigma^*_Q(\Omega^*_Q), \frac{1}{2}^+\to\frac{3}{2}^+$ transitions

$$<\Sigma^*_Q(P_2)|J^{V-A}_\lambda|\Sigma_Q(P_1)>= b(\Sigma_Q,\Sigma^*_Q)\bar{u}^\mu_2\gamma_\lambda(1-\gamma_5)(\gamma^\nu+v^\nu_1)\gamma_5u_1(P_1)\tilde{L}_{\mu\nu} \tag{4.10}$$

iv) $\Sigma^*_Q(\Omega^*_Q)\to\Sigma^*_Q(\Omega^*_Q)$-type $\frac{3}{2}^+\to\frac{3}{2}^+$ transition

$$<\Sigma^*_Q(P_2)|J^{V-A}_\lambda|\Sigma^*_Q(P_1)>= b(\Sigma^*_1,\Sigma^*_2)\bar{u}^\mu_2(P_2)\gamma_\lambda(1-\gamma_5)u^\nu_1(P_1)\tilde{L}_{\mu\nu} \tag{4.11}$$

So if the heavy diquark picture is a good approximation, all the heavy to heavy baryon transitions are characterized by only one form factor $\tilde{F}(\omega) = 8A_1(\omega)D_1(\omega)$. Comparing with the results given in Sec.II, we come to the conclusion that the presence of heavy diquark reduces the number of the weak form factor from 3 to 2. The existence of the heavy quark is checked by the relations

$$F_1(\omega) = \omega\tilde{F}(\omega)$$
$$F_2(\omega) = F_\Lambda(\omega) = \tilde{F}(\omega) \tag{4.12}$$

or

$$\omega F_2(\omega) = F_1(\omega) = \omega F_\Lambda(\omega) \tag{4.13}$$

Checking the equalities (4.14) in the heavy to heavy baryon transitions we can be able to verify or exclude the heavy diquark picture. The relations between weak form factors in this section have been obtained in the work [7] based on the assumption of "independent light quarks". According to our interpretations given previously, it is the case of the heavy diquark.

V. THE HEAVY TO LIGHT TRANSITIONS IN THE HEAVY DIQUARK MODEL OF 1HQ-BARYONS

There are two cases represented by the Feynman graphs in Fig.7 and Fig.8. Let us consider these cases separately

A. *The case of stable diquark*

The Feynman graph in Fig.7 corresponds to this case, where the decaying diquark remains stable. In this case the correlation inside the diquark remains strong to keep these quarks together. The produced quark cannot be lighter than the spectator quark according to the idea of heavy diquark.

footnotesize Fig.7

Using the diquark-quark-baryon B-S wave functions of heavy baryons and light baryons given in Sec.III. Let us consider the k_2-dependent part of the light baryon wave functions

$$A^{\rho\delta}_{\alpha\beta}(v,k_2) = A^{[\rho\delta]}_{\alpha\beta}(v,k_2) + A^{(\rho\delta)}_{\alpha\beta} \tag{5.1}$$

The symmetric part $A^{(\rho\delta)}_{\alpha\beta}$ can be written as follows:

$$A^{(\rho\delta)}_{\alpha\beta} = A^{1}_{\alpha\beta}(C^{-1})^{\rho\delta} + A^{2}_{\alpha\beta}(C^{-1}\gamma_5)^{\rho\delta} + A^{3\mu}_{\alpha\beta}(C^{-1}\gamma_\mu\gamma_5)^{\rho\delta} \tag{5.2}$$

The antisymmetric part is

$$A^{[\rho\delta]}_{\alpha\beta} = \tilde{A}^{1\mu}_{\alpha\beta}(C^{-1}\gamma_\mu)^{\rho\delta} + \tilde{A}^{2\mu\nu}_{\alpha\beta}(c^{-1}\sigma_{\mu\nu})^{\rho\delta} \tag{5.3}$$

Inserting the expressions (5.2) and (5.3) into the wave function of light baryons, we can compute the following decay matrix elements

i) $\Lambda_Q(\Xi_Q) \rightarrow \Lambda_q(\Xi_q), \frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ baryon transitions

$$
\begin{aligned}
< \Lambda_q(P_2)|J_\lambda^{V-A}|\Lambda_Q(P_1) > &= b(\Lambda_q, \Lambda_Q)\bar{u}_2^\rho(P_2)[\gamma_\lambda(1-\gamma_5)u_1(P_1)]_\alpha \hat{F}_\rho^\alpha \\
&= \bar{u}_2(P_2)(F_1^\Lambda(\omega) + \not{v}_1 F_2^\Lambda(\omega))\gamma_\lambda(1-\gamma_5)u(v_1) \qquad (5.4)
\end{aligned}
$$

where

$$
\hat{F}_\rho^\alpha = [(\not{v}_2+1)\gamma_5 C]^{+\delta\sigma}[(\not{v}_1+1)\gamma_5 C]_{\delta'\sigma}\hat{A}^{\alpha\delta'}_{[\rho\delta]}D_1(\omega) \qquad (5.5)
$$

and

$$
\hat{A}^{\alpha\delta'}_{[\rho\delta]} = \int d^4k_2 A^{+\alpha\beta}_{\ \ \rho\delta}(v_2,k_2)(\not{k}_2 - m_2)^{\beta'}_\beta A^{\delta'}_{\beta'}(v_1,v_1) \qquad (5.6)
$$

There is no collapsing of the form factors. However, the loop integral is greatly simplified.

ii) $\Sigma_Q(\Omega_Q) \rightarrow \Sigma_q(\Omega_q), \frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ transitions

$$
\begin{aligned}
< \Sigma_q(P_2)|J_\lambda^{V-A}|\Sigma_Q(P_1) > &= b(\Sigma_Q, \Sigma_q)[\bar{u}_2(P_2)\gamma_5(\gamma^\mu + v_2^\mu)]^\rho[\gamma_\lambda(1-\gamma_5)(\gamma^\nu \\
&+ v_1^\nu)\gamma_5 u_1(P_1)]_\alpha[(\not{v}_2+1)\gamma_\mu C]^{+\delta\sigma}[(\not{v}_1+1)\gamma_\nu C]_{\delta'\sigma}\tilde{A}^{\alpha\delta'}_{(\rho\delta)}D_1(\omega) \\
&= \bar{u}_2(P_2)(\not{v}_1+1)\gamma_\nu\gamma_5(\tilde{G}_1(\omega) + \tilde{G}_2(\omega)\not{v}_2)\gamma_\lambda(1-\gamma_5)(\gamma^\nu + v_1^\nu)\gamma_5 u_1(P(1) \qquad (5.7)
\end{aligned}
$$

where

$$
\tilde{A}^{\alpha\delta'}_{(\rho\delta)} = \int d^4k_2 A^{+\alpha\beta}_{\ \ (\rho\delta)}(v_2,k_2)(\not{k}_2 - m_2)^{\beta'}_\beta A^{\delta'}_{\beta'}(v_1,k_2) \qquad (5.8)
$$

Comparing with the formulas given in Sec.II we obtain the relations

$$
F_1(\omega) = -\tilde{G}_1(\omega) - \tilde{G}_2(\omega) - 2\omega \qquad (5.9)
$$

$$
F_2(\omega) = -2\tilde{G}_2(\omega) \qquad (5.10)
$$

$$
F_3(\omega) = \tilde{G}_2(\omega) - \tilde{G}_1(\omega) \qquad (5.11)
$$

$$
F_4(\omega) = 2\tilde{G}_2(\omega) \qquad (5.12)
$$

The relations to be verified are

$$
F_2(\omega) = -F_4(\omega) \qquad (5.13)
$$

$$
F_1(\omega) = F_3(\omega) - F_4(\omega) - 2\omega \qquad (5.14)
$$

Four form factors collapse to two.

iii) $\Sigma_Q(\Omega_Q) \to \Sigma_q^*(\Omega_q^*)$, $\frac{1}{2}^+ \to \frac{3}{2}^+$ transitions

$$< \Sigma_q^*(P_2)|J_\lambda^{V-A}|\Sigma_Q(P_1) >= b(\Sigma_Q, \Sigma_q^*)(\bar{u}^\mu)_2^\rho\Big[\gamma_\lambda(1-\gamma_5)(\gamma^\nu + v_1^\nu)$$
$$\gamma_5 u_1(P_1)\Big]^\alpha[(\not{v}_2+1)\gamma_\mu C]^{+\delta\sigma}[(\not{v}_1+1)\gamma_\mu C]_{\delta'\sigma}\tilde{A}^{\alpha\beta}_{(\rho\delta)}(v_1,v_2)D_1(\omega) \quad (5.15)$$
$$= \; b(\Sigma_q, \Sigma_Q^*)\bar{u}_2^\mu(P_2)\gamma_\nu[v_{1\mu}H_1(\omega) + H_2(\omega)v_{1\mu}\not{v}_2]\gamma_\lambda(1-\gamma_5)(\gamma^\nu$$
$$+ \; v_1^\nu)\gamma_5 u_1(P_1) \quad (5.16)$$

Comparing with the formulas in Sec.II. we obtain the relations

$$G_1^*(\omega) = 0$$
$$G_2^*(\omega) = 2H_2(\omega)(2\omega - 1)$$
$$G_3^*(\omega) = 0$$
$$G_4^*(\omega) = -2H_2(\omega)$$
$$G_5^*(\omega) = (1-2\omega)H_2(\omega) - H_1(\omega)$$
$$G_6^*(\omega) = H_1(\omega) + H_2(\omega) \quad (5.17)$$

The number of the form factors has also reduced to 2.

iv) $\Sigma_Q^*(\Omega_Q^*) \to \Sigma_Q^*(\Omega_Q^*)$-type $\frac{3}{2}^+ \to \frac{3}{2}^+$ transition

$$< \Sigma_Q^*(P_2)|J_\lambda^{V-A}|\Sigma_Q^*(P_1) >= b(\Sigma_1^*, \Sigma_2^*)(\bar{u}_2^\mu)^\rho(P_2)[\gamma_\lambda(1-\gamma_5)u_1^\nu(P_1)]_\alpha$$
$$[(\not{v}_2+1)\gamma_\mu C]^{+\delta\sigma}[(\not{v}_1+1)\gamma_\nu C]_{\delta'\sigma}\tilde{A}^{\alpha\delta'}_{(\rho\delta)}D_1(\omega)$$
$$= \; \bar{u}_2^\mu(v_2)\gamma^\nu(\not{v}_1 - 1)v_{1\mu}(H_1(\omega) + H_2(\omega)\not{v}_2)\gamma_\lambda(1-\gamma_5)u_1(P_1) \quad (5.18)$$

We have also two form factors H_1 and H_2 in this case

B. *The case of new light diquark forming in the produced baryon*

This case is represented by the Feynmann graph in Fig.8.

Fig.8

In this case the new quark becomes lighter than the spectator one, so the diquark becomes unstable and one of its quark leaves the boundstate to join the other quark forming a new light diquark. Inserting the wave functions of light baryons and heavy baryons into the formula (2.12) we see that as the loop integrals are not separated no collapsing of the form factors occurs as consequence of the diquark picture.

VI. DISCUSSION

Let us summarize the results before going further. The suggestion that the heavy diquark may exist inside the 1HQ-baryons leads to some constraints between the weak form factors in some cases. It is remarkable that the concept of diquark used in this paper is after Lichtenberg[LICH]. That means, we have not postulated more than to suppose that the correlation between two quarks is much stronger than any other. No local elementary diquark field has been assumed. The analysis here has been based on the fully antisymmetrized wave function of baryons. In the heavy to heavy baryon transitions the number of form factors reduces from 3 to 1 as the consequence of heavy diquark hypothesis. In the heavy to light baryon transitions if the produced quark can still keep its partner the number of the form factors reduces greatly. The stability of the decaying diquark could have different reasons. In our opinion if the produced quark is still heavier than the quark outside the diquark it will be the case. Anyway, the constraints on the weak form factors of these transitions would testify which diquark exists in the produced baryons.

If in the heavy baryon a point-like heavy diquark does exist we should be able to indicate its existence by the structure of form factors. Assuming that such a point-like structure does exit inside baryons, we can treat the diquark as an elementary local field. As the diquark is a boson and very heavy, in the limit of the heavy quark mass going to infinity we can construct an effective theory for heavy diquark following Georgi, Wise and Carone [9]. Now the heavy boson of Georgi, Wise and Carone has a new interpretation as the heavy diquark.

It is interesting to note that in such an effective theory in the heavy quark mass limit, there is a supersymmetry between the heavy diquark and the heavy antiquark. If this supersymmetry survives the hadronic level, we can use the Wigner-Eckart theorem to derive the same formula as the one of Georgi and Carone [9] for the matrix element:

$$< \Lambda_Q(P_2)|J^V_\lambda|\Lambda_Q(P_1) >= b(\Lambda_q, \Lambda_Q)\bar{u}_2(v_2)u_1(v_1)\xi(\omega)(v_{1\mu} + v_{2\mu}) \tag{6.1}$$

The $\xi(\omega)$ form factors is sometimes called Isgur-Wise function. This result is quite different from the formula (2.13). By the help of the Gorkov identity we can transform the formula (6.1) into another form with the electric -type term containing γ_λ and a magnetic term containing $\sigma_{\lambda\mu}(v_2^\mu - v_1^\mu)$. Comparing with the equation (2.13), we see that an elementary heavy diquark structure leads to a magnetic form factor, while the usual HQET does not. The explanation is simple: the magnetic moment of the baryon with an elementary diquark comes from the spin of the light quark. With light diquark the magnetic moment of the baryon comes from the spin of the heavy quark, which contributes nothing to the weak form factors due to the heavy quark symmetry.

It is also interesting to note that the heavy diquark picture give the same number of form factors for mesons and baryons. This would mean that the supersymmetry can occur. Originally, supersymmetry of hadrons was proposed by Miyazawa [10] to derive some similarities between baryons and mesons as early as in 1968. Recently Catto and Gürsey [11]found QCD basics for this symmetry and explained the parallelism between mesonic and baryonic Regge trajectories. As the supersymmetry is broken badly for light hadrons, Lichtenberg [12] argued that the supersymmetry will become good for heavy hadrons. There are two types of supersymmetry depending on the existence of diquark. The supersymmetry of Lichtenberg is associated with the light diquark. The second supersymmetry $SU(1/6)$ is associated with the heavy quark. So far, the supersymmetry in the world of 1HQ-baryons occurs on two basic assumptions : the existence of diquark and the heavy quark symmetry. It seems too strict to suppose so much. But the nature is always simpler than we thought. The question is whether the heavy diquark picture is good and on which energy scale. It is very likely

that the heavy diquark does exist on the same energy scale as the one of the heavy quark symmetry or even lower than that. If this is true we can expect that the supersymmetry occurs at the same time as the heavy quark symmetry.

Presently, we don't have much informations about weak decays of heavy baryons. Hopefully, more data of heavy baryon's weak decays will be available in the near future to clarify our questions: Which diquark picture (heavy or light) is the better approximation to 1HQ baryons? Can diquark be approximated as a point-like object? Is there any supersymmetry between 1HQ hadrons? Basing on our results a future analysis on the weak decay data of baryons will determine whether these postulations can make any sense.

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[*] Permanent address: Centre for Theoretical Physics, P.O.Box 429, Bo Ho 429, Hanoi, Vietnam.

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FIGURES

Fig.1 The quark-baryon Bethe-Salpeter amplitude.

Fig.2 The 1HQ baryon-quark Bethe-Salpeter amplitude.

Fig.3 The weak current induced baryon transitions.

Fig.4 The ansatz for the light baryon Bethe-Salpeter amplitudes

Fig.5 The heavy diquark-quark-baryon Bethe-Salpeter amplitude

Fig.6 The heavy to heavy current induced baryon transition in the heavy diquark model

Fig.7 The heavy to light baryon transition in the heavy diquark model. The diquark remains stable after the weak decay of the heavy quark

Fig.8 The heavy to light baryon transition in the heavy diquark model. A new diquark appears inside the light baryon

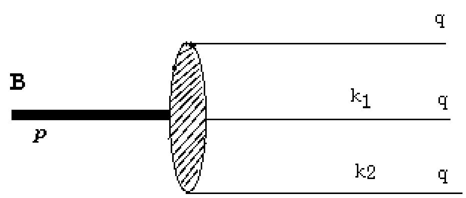

Fig. 1

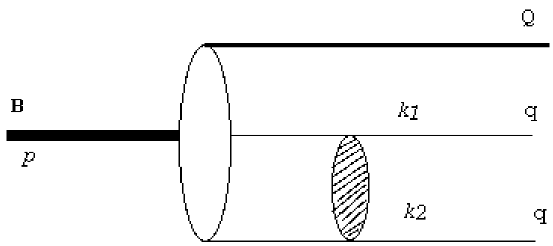

Fig.2

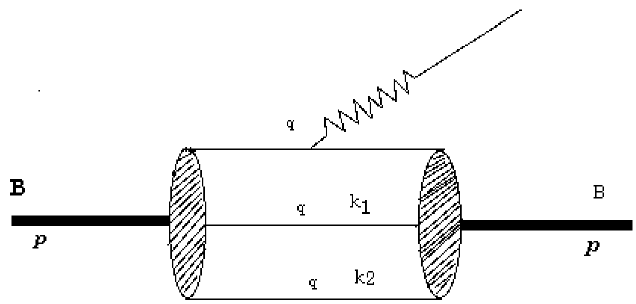

Fig. 3

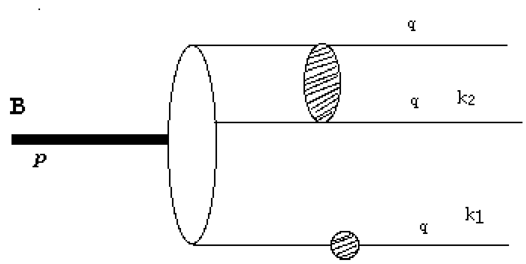

Fig.4

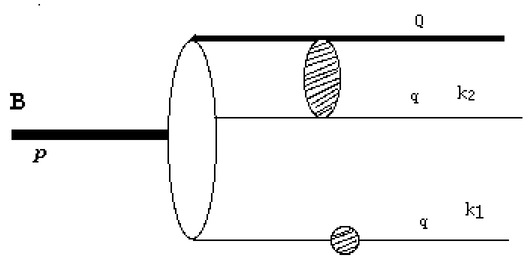

Fig.5

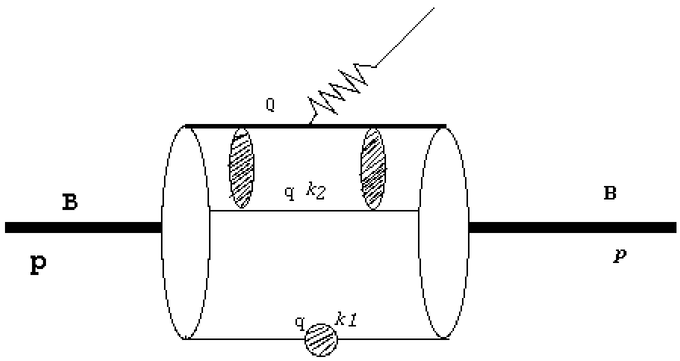

Fig. 6

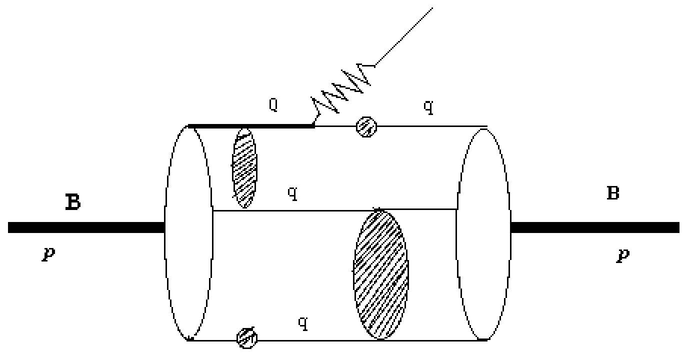

Fig. 7

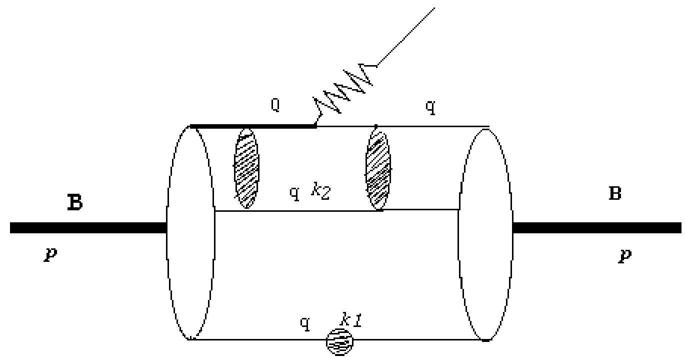

Fig. 8